Phase-controlled proximity effect in ferromagnetic Josephson junctions: Calculation of the density of states and the electronic specific heat

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We study the thermodynamic properties of a dirty ferromagnetic S|F|S Josephson junction with s-wave superconducting leads in the low-temperature regime. We employ a full numerical solution with a set of realistic parameters and boundary conditions, considering both a uniform and nonuniform exchange field in the form of a Bloch domain wall. Our main result is that it is possible to *strongly modify the electronic specific heat* of the system by changing the phase difference between the two superconducting leads from zero up to nearly π. This is explained in terms of the proximity-altered density of state in the ferromagnetic region, and we discuss possible methods for experimental detection of the predicted effect.

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I. INTRODUCTION

In recent years, due to the important role of hybrid structures with superconducting and magnetic layers in vital circuit elements such as transistors and high-resolution devices like detectors, such structures are intensely investigated due to their potential both in terms of functionality[1] and novel fundamental physics that may be explored.[2,3] In the context of applications, studies of the thermodynamic properties of superconductors have mostly focused on electron cooling properties,[4] although recently the influence of the proximity effect on the entropy production in a nonmagnetic Josephson junction was investigated.[5] A key to understanding the thermodynamic properties of a system is the behavior of the density of states (DOS) near Fermi level, and any control parameter that can adjust the DOS in an efficient and well-defined manner would offer significant advantages with respect to tailoring desired thermodynamic properties.

Very recently, it has been studied numerically[6] and demonstrated experimentally[7] how the DOS may be altered controllably in such structures by creating a nonmagnetic S|N|S Josephson junction and generating a supercurrent. The phase dependence of the mini gap[8] was experimentally investigated using scanning tunneling microscopy-atomic force microscopy and found to agree well with theoretical predictions. For a ferromagnetic Josephson junction, however, it remains to be clarified precisely how the DOS is influenced by the phase difference in the full proximity effect regime. Two features which are expected to come into play for ferromagnetic Josephson junctions are the presence of odd-frequency superconducting correlations, which can induce a qualitative shift in the DOS from a low-energy mini gap-structure[8] to an enhancement, and spin-active interfaces.[9,10]

In this paper, we show how it is possible to obtain a *huge enhancement of the specific heat* of a ferromagnetic Josephson junction at low temperatures, simply by tuning the superconducting phase difference by means of either a current or an external magnetic flux in a superconducting quantum interference device (SQUID)-like geometry. We demonstrate explicitly how the predicted effect occurs for a set of realistic experimental parameters and how it persists even in the presence of an inhomogeneous magnetization texture such as a Bloch domain wall in the ferromagnet. We find that the enhancement of the specific heat is strongest for exchange fields h comparable in magnitude to the superconducting gap Δ, i.e., $h \simeq \Delta$, whereas for higher exchange fields the effect eventually vanishes. Our findings can be verified experimentally by using calorimetry techniques[11] or high-resolution thermometry.[12] We underline that while it is well known that the DOS in a Josephson junction is sensitive to the phase difference, our main result pertains to the manner in which the DOS varies and the resulting consequences for the electronic specific heat of the junction. While this has been studied previously for a normal Josephson junction, it has to the best of our knowledge not been investigated for a ferromagnetic superconductor|ferromagnet|superconductor (S|F|S) junction. The findings of Ref. 7 demonstrate the feasibility of experimentally verifying the results for the DOS in this work, whereas our predictions for the specific heat can be tested directly via the route employed in Ref. 11 involving calorimetry measurements in a SQUID loop geometry with an array of Josephson junctions.

II. THEORY

To investigate the physical properties of the S|F|S Josephson junction, we solve the quasiclassical equations of superconductivity[13,14] to obtain the Green's functions. It is convenient to use a Ricatti parametrization of the latter as follows:[6,15,16]

$$\hat{g} = \begin{pmatrix} \underline{\mathcal{N}}(\underline{1} - \underline{\gamma}\underline{\tilde{\gamma}}) & 2\underline{\mathcal{N}}\underline{\gamma} \\ 2\underline{\tilde{\mathcal{N}}}\underline{\tilde{\gamma}} & \underline{\tilde{\mathcal{N}}}(-\underline{1} + \underline{\tilde{\gamma}}\underline{\gamma}) \end{pmatrix}. \tag{1}$$

Here, $\hat{g}^2 = \hat{1}$ since $\underline{\mathcal{N}} = (1 + \underline{\gamma}\underline{\tilde{\gamma}})^{-1}\underline{\tilde{\mathcal{N}}} = (1 + \underline{\tilde{\gamma}}\underline{\gamma})^{-1}$. We use $\underline{\ldots}$ for 2×2 matrices and $\hat{\ldots}$ for 4×4 matrices. An alternative parametrization procedure in the context of inhomogeneous ferromagnets was employed in Ref. 17. In order to calculate the Green's function $\hat{g}$, we need to solve the Usadel equation[14] with appropriate boundary conditions at $x = -d_F/2$ and $x = d_F/2$. We introduce the superconducting coherence length as $\xi_S=\sqrt{D_S/\Delta_0}$. Following the notation of Ref. 18, the Usadel equation reads $D\partial(\hat{g}\partial\hat{g})+i[E\hat{\rho}_3+\text{diag}[\boldsymbol{h}\cdot\underline{\boldsymbol{\sigma}},(\boldsymbol{h}\cdot\underline{\boldsymbol{\sigma}})^T],\hat{g}]=0$, and we employ the following realistic boundary conditions for all our computations in this paper:[9] $2\zeta d_F\hat{g}\partial\hat{g}=[\hat{g}_{\text{BCS}}(\phi),\hat{g}]+i(G_S/G_T)[\text{diag}(\underline{\tau_3},\underline{\tau_3}),\hat{g}]$ at $x=-d_F/2$. Here, $\partial\equiv\frac{\partial}{\partial x}$ and we defined $\zeta=R_B/R_F$ as the ratio between the resistance of the barrier region and the resistance in the ferromagnetic film. The barrier conductance is given by G_T, whereas the parameter G_S describes the spin-dependent interfacial phase shifts (spin DIPS) taking place at F side of the interface where the magnetization is assumed to lie in the yz plane, being parallel to the z axis at the interfaces. The boundary condition at $x=d_F/2$ is obtained by letting $G_S\to(-\tilde{G}_S)$ and $\hat{g}_{\text{BCS}}(\phi)\to[-\hat{g}_{\text{BCS}}(-\phi)]$ in the boundary condition at $x=-d_F/2$, where $\underline{\gamma}_{\text{BCS}}(\phi)=i\underline{\tau_2}s/(1+c)e^{i\phi/2}$, $\underline{\tilde{\gamma}}_{\text{BCS}}(\phi)=\underline{\gamma}_{\text{BCS}}(\phi)e^{-i\phi}$. Above, $\tilde{G}_S$ is allowed to be different from G_S in general. For instance, if the exchange field has opposite direction at the two interfaces due to the presence of a domain wall, one finds $\tilde{G}_S=-G_S$. The total superconducting phase difference is ϕ, and we have defined $s=\sinh(\vartheta)$, $c=\cosh(\vartheta)$ with $\vartheta=\text{atanh}(\Delta_0/E)$ using Δ_0 as the superconducting gap. Note that we use the bulk solution in the superconducting region, which is a good approximation when assuming that the superconducting region is much less disordered than the ferromagnet and when the interface transparency is small, as considered here. Effectively, the inverse proximity effect is thus ignored. We use units such that $\hbar=k_B=1$. The values of G_S and G_T may be calculated explicitly from a microscopic model, which allows one to characterize the transmission $\{t_{n,\sigma}^j\}$ and reflection amplitudes $\{r_{n,\sigma}^j\}$ on the $j\in\{S,F\}$ side.[9,10,19]

To find the specific heat of the system, we need to calculate the local density of states normalized against its normal-state value $N(x,E,T,\phi)=\text{Tr}\{\text{Re}[\underline{\mathcal{N}}(\underline{1}-\underline{\gamma}\underline{\tilde{\gamma}})]\}/2$. We assume that the S electrodes are not influenced by the proximity effect; the total electronic specific heat (C_{tot}) of the S|F|S junction can be determined by $C_{\text{tot}}(T,\phi)=C_F(T,\phi)+C_S(T)$. Here, $C_S(T)$ is the specific heat of superconducting plates while $C_F(T,\phi)=T\partial S_F(T,\phi)/\partial T$ is the specific heat of the ferromagnetic part of the junction. The entropy of the ferromagnet layer in the proximity system can be obtained from $S_F(T,\phi)=-(4/L)\int_{-L/2}^{L/2}dx\int_0^\infty dE\, N(x,E,T,\phi)\{f(E)\ln[f(E)]+[1-f(E)]\ln[1-f(E)]\}$, and $f(E)=\{1+\exp[E/T]\}^{-1}$ is the Fermi-Dirac distribution function at temperature T.

Since we employ a numerical solution, we have access to study the full proximity effect regime and also, in principle, an arbitrary spatial modulation $h=h(x)$ of the exchange field. This is desirable in order to clarify effects associated with nonuniform ferromagnets, such as the presence of Bloch domain walls. Here, we will consider two different types of magnetization textures: homogeneous magnetization and a Bloch domain-wall structure. The Bloch model is given by $\mathbf{h}=h(\cos\theta\hat{y}+\sin\theta\hat{z})$, where $\theta=-2\arctan(x/d_W)$,[16] with d_W as the width of domain wall. Moreover, the center of the F layer is located at the origin $x=0$. Below, we shall consider a domain wall of width $d_W/d_F=0.5$, thus ensuring that the magnetization is fully aligned with the z axis at the interfaces. Our model for the domain wall is standard in the literature and contains the essential ingredients of the spatial magnetization texture expected for such a topological defect. Having stated this, it is certainly an experimental issue to determine quantitatively, e.g., the length scales associated with this texture for appropriate ferromagnetic materials employed in S|F|S junctions.

III. RESULTS AND DISCUSSION

We now present our main results of the paper. In the quasiclassical framework employed here, we have to consider an exchange field much weaker than the Fermi energy in order to remain within the regime of validity. For a weak diffusive ferromagnetic alloy such as PdNi, the exchange field h can be varied from a few meV to tens of meV by changing the relative contents of Pd and Ni. Even weaker exchange fields h of order meV are found in, for instance, Y_4Co_3, Y_9Co_7, and $TiBe_{1.8}Cu_{0.2}$.[20] Therefore, we shall consider here exchange fields ranging from 0.5 meV up to 5 meV. The scenario of a thin junction $d_F/\xi_S=0.3$ will be contrasted with that of a thick junction $d_F/\xi_S=1.0$. For a superconducting lead like Nb with a coherence length of $\xi_S\simeq 18$ nm, the ratio of $d_F/\xi_S=0.3$ provides a ferromagnetic layer thickness equal to 6 nm, which is experimentally accessible.[21] The temperature will be fixed at $T=0.05T_c$ and consequently our results are valid for low-temperature regime. The spin-dependent interfacial phase-shift (spin-DIPS) term G_S is obtained via the microscopic theory introduced in the previous section and depends, e.g., on the magnitude of the exchange field and the interface transparency. Since we calculate G_S microscopically for a simplified model with a Dirac tunneling barrier, G_S is not treated as a phenomenological parameter here. We choose $\mu_F=1$ eV and $\mu_S=10$ eV for the Fermi level in the ferromagnet and superconductor, respectively, and consider a relatively low transparency barrier of $Z_0=3$. The electron mass m_F and m_S in both of the F and S regions is taken to be the bare one ($\simeq 0.5$ MeV). The ratio of the electronic resistances of the barrier region and the ferromagnet layer is assumed to be $\zeta=R_B/R_F=4$ throughout our computations. We also insert a small imaginary part $\delta=10^{-3}\times\Delta_0$ into quasiparticle energies, i.e., $E\to E+i\delta$ for access to more stability in our computations. The small imaginary part can be interpreted as accounting for inelastic scattering. As we discuss below, we find that the specific heat of the S|F|S diffusive junction can be strongly enhanced by changing the phase difference between the two singlet superconducting leads from zero up to values near π for both a homogeneous exchange field and in the domain-wall case. Due to limitations of our numerical code, we were not able to investigate phase differences ϕ very close to π. The huge enhancement of the specific heat can be seen even for values of the exchange field several times the superconducting gap in the domain-wall case. Upon increasing the magnitude of the exchange field further to values $h\gg\Delta_0$, this effect vanishes. The enhancement is most resilient toward an increase in h in the case where a domain wall is present. Both the enhancement of the specific heat and its persistence in the domain-wall case can be understood by investigating the DOS in the ferromagnetic region. We now proceed to a presentation of our main results.

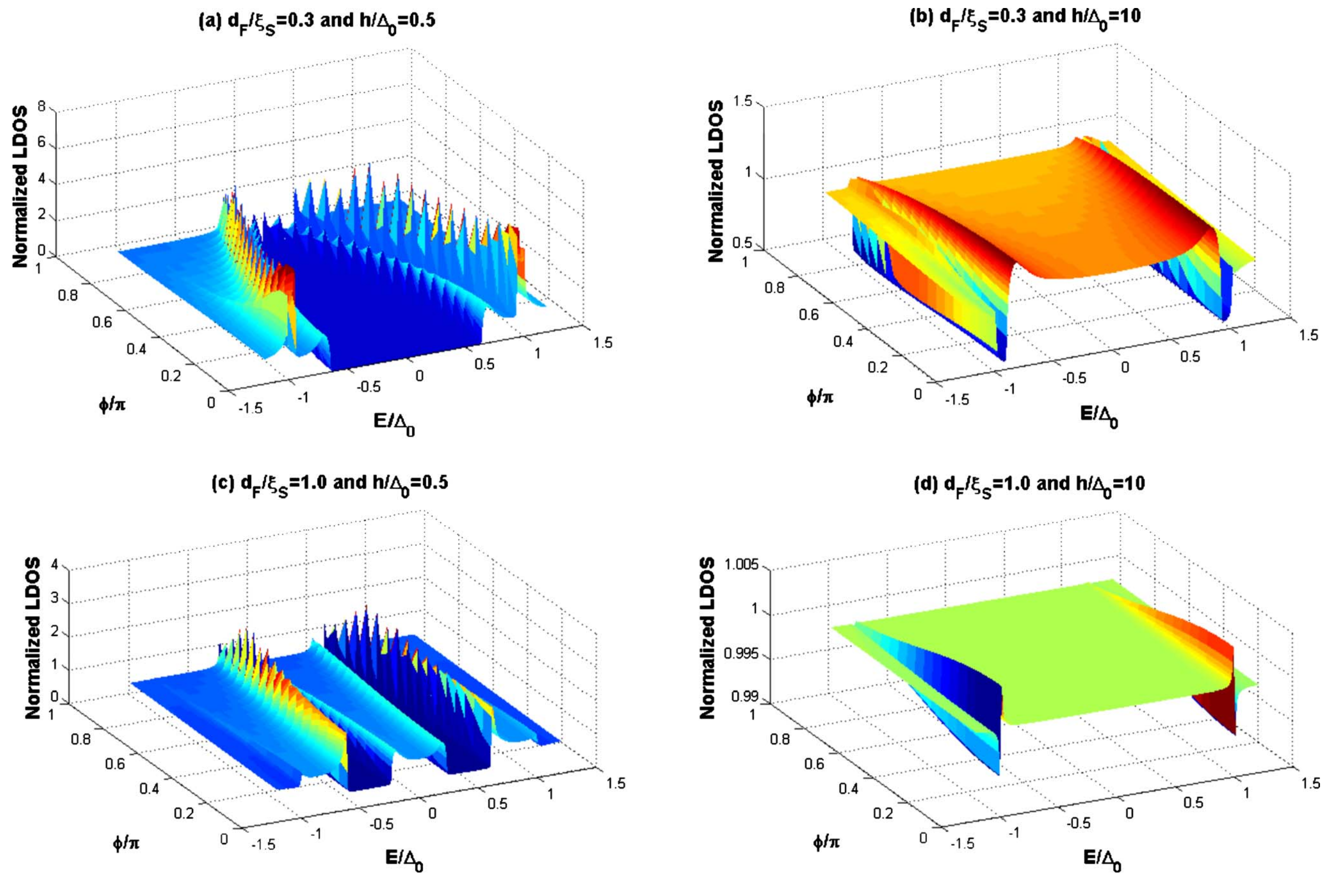

FIG. 1. (Color online) The normalized local density of states vs energy and phase difference between two *s*-wave superconducting leads for a uniform magnetization texture with (a) $d_F/\xi_S=0.3$ and (b) $d_F/\xi_S=1.0$. The DOS is evaluated in the middle of the junction, i.e., $x=0$, and $h/\Delta_0=0.5$.

Let us start by discussing the behavior of the DOS in a ferromagnetic region by changing the phase difference between two superconducting leads connected to it. The results are shown in Fig. 1 for the homogeneous exchange field case, while the domain-wall scenario is demonstrated in Fig. 2. In both figures, we provide a contour plot of the DOS in the middle of the F layer as a function of quasiparticle energy E measured from Fermi level and the superconducting phase difference ϕ. In both Figs. 1 and 2, $h/\Delta_0=0.5$. Let us first consider the homogeneous case shown in Fig. 1(a) for a thin ferromagnetic layer $d_F/\xi_S=0.3$. The most obvious feature is that a mini gap structure is induced in the low-energy regime close to the Fermi level, flanked by a peak structure below the gap and at the gap. The mini gap is shown to close as the phase difference moves toward $\phi=\pi$, as is also the case for S|N|S junctions.[6] In Fig. 1(b), the junction thickness is increased to $d_F/\xi_S=1.0$, and it is seen that the peak structures remain. The main difference from Fig. 1(a) is that the low-energy DOS is enhanced, indicating that the odd-frequency correlations are present and comparable in magnitude to the even-frequency correlations. The mini gap is split into two and is seen to shift away from zero energy. The appearance of the multiple peak structures as a function of energy E originates from an effective superconducting gap felt by each spin species which is different in magnitude for spin-$\uparrow$ and spin-$\downarrow$ quasiparticles. This is similar to the scenario of thin-film superconductors subjected to an in-plane external magnetic field.[22]

We now turn to the domain-wall case, shown in Fig. 2. The most noteworthy change from Fig. 1 is that the zero-energy DOS is enhanced in (b). This observation signals that odd-frequency correlations are stronger in the domain-wall case, a finding which agrees with the results in Ref. 18. The physical reason for this is that the inhomogeneous magnetization texture generates not only the $S_z=0$ triplet component, but also the long-ranged $S_z=\pm 1$ triplet components, which also are odd in frequency due to the isotropization caused by the impurity scattering. The presentation of the DOS and its dependence on the energy E and superconducting phase difference ϕ presented here is a useful preliminary which, as we shall see, explains the origin behind our main result of a strongly enhanced specific heat, which we shall now move on to.

Let us then consider the electronic specific heat of the S|F|S diffusive Josephson junction vs phase difference of the two superconducting leads. As mentioned in the Introduction, the phase difference is an experimentally tunable quantity by means of, e.g., current biasing the junction or applying an external magnetic field in a SQUID-like geometry. Since the proximity effect is in general much weaker in the low-energy regime for $d_F/\xi_S=1.0$, we focus here on the more interesting case $d_F/\xi_S=0.3$.

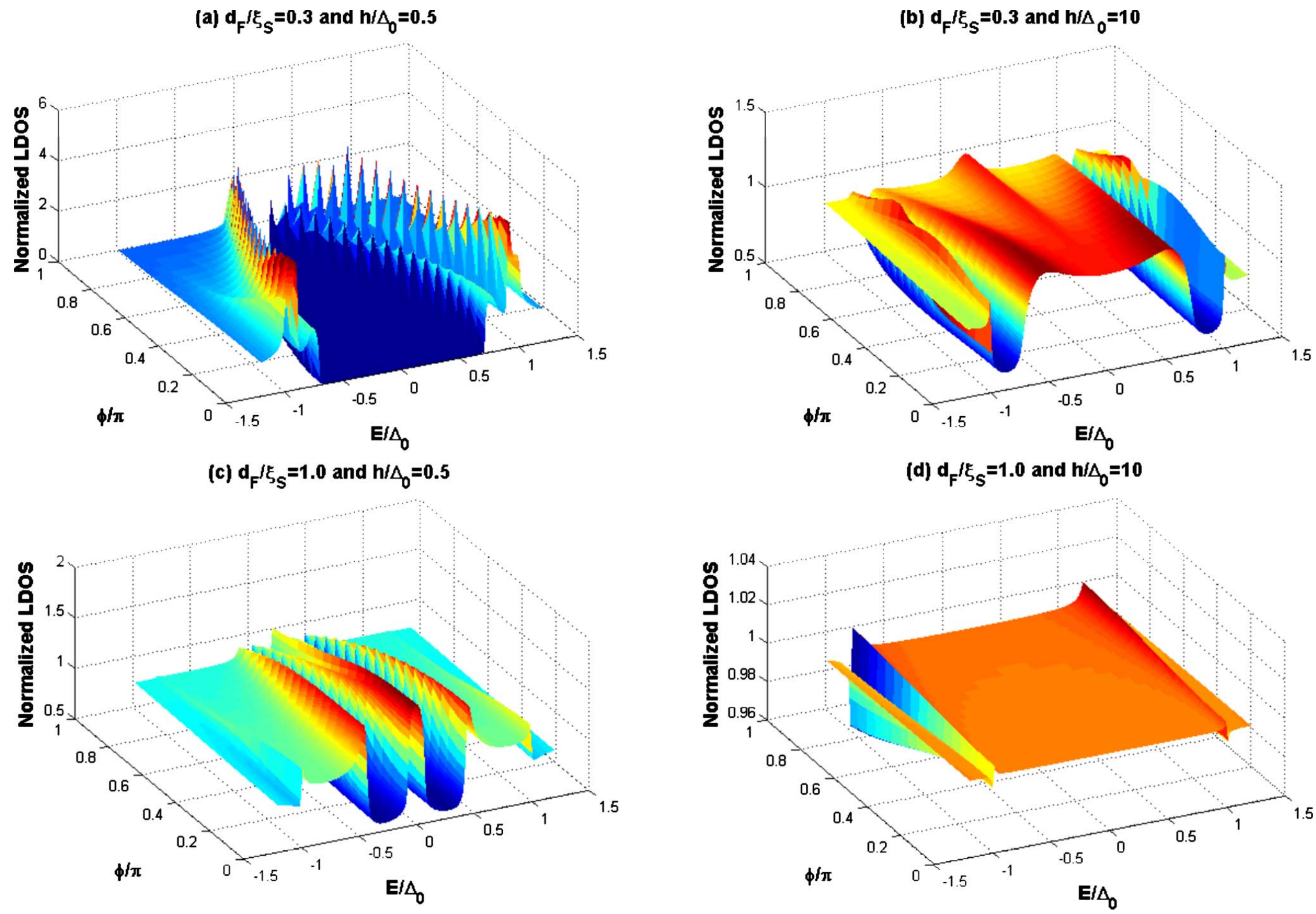

FIG. 2. (Color online) The normalized local density of states vs energy and phase difference between two s-wave superconducting leads and inhomogeneous magnetization texture for (a) d_F/ξ_S=0.3 and (b) d_F/ξ_S=1.0. The DOS is evaluated in the middle of the junction, i.e., x=0, and h/Δ_0=0.5.

The results are shown in Fig. 3. As seen, the left panel is related to the homogeneous exchange field scenario while the right panel is related to the inhomogeneous magnetization in the form of a Bloch domain wall. In both cases, the curves show a giant enhancement of the normalized specific heat when the exchange field is comparable in magnitude to the superconducting gap. For larger exchange fields, the specific heat becomes a monotonic nearly constant function of the phase difference ϕ. We note that the enhancement persists for larger values of h in the domain-wall case (up to $h/\Delta_0 \simeq 3.0$) compared to the homogeneous case. The physical reason behind the enhancement of the specific heat stems

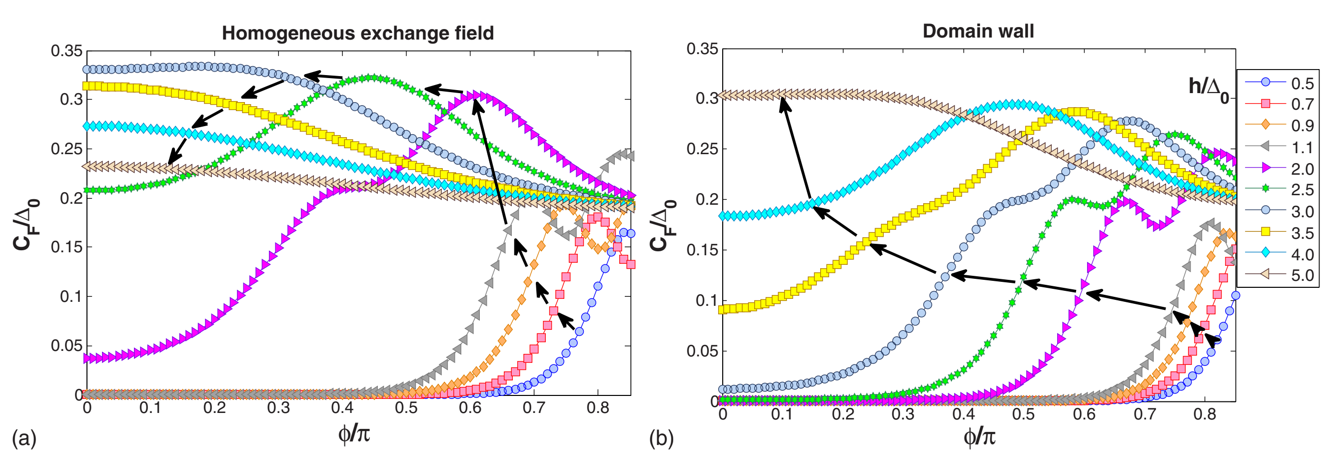

FIG. 3. (Color online) The normalized electronic specific heat of the diffusive S|F|S Josephson junction vs phase difference between two s-wave superconducting leads with a ferromagnetic layer featuring a homogeneous and an inhomogeneous Bloch domain-wall magnetization texture. The arrows indicate an increasing value of h.

from the dependence of the DOS on ϕ, as shown in Figs. 1 and 2. For instance, for a very weak exchange field $h/\Delta_0=0.5$, the DOS plots presented in the previous section showed how the mini gap closed with increasing ϕ. Since it is the low-energy DOS that mainly contributes to the electronic specific heat, increasing the phase difference ϕ will naturally lead to an increase in C_F. More specifically, we have verified numerically that at $T/T_c=0.05$, only energies up to $E/\Delta_0\simeq 0.35$ contribute to the specific heat integral.

By increasing the magnitude of the exchange field in the ferromagnetic layer, a kink appears in the specific heat. To identify the cause of the appearance of the kinks, one should investigate the related DOS of the system. Consider now for concreteness the DOS in the homogeneous exchange field case with $h/\Delta_0=1.1$, which is seen to display a kink in the specific heat in the left panel of Fig. 3. The kink of this curve appears near $\phi/\pi\simeq 0.7$, consequently leading us to plot the DOS of the system near this value vs E/Δ_0 and ϕ/π. The resulting DOS is shown in Fig. 4. As seen, the cause of the appearance of a kink near $\phi/\pi\simeq 0.7$ is the zero-energy peak that occurs in this region of the phase difference. Such a zero-energy peak should be a direct result of the manifestation of odd-frequency correlations in the system.[23] In Fig. 4, it is seen that an abrupt conversion takes place at $\phi/\pi\simeq 0.7$ along the $E=0$ line from a fully suppressed DOS to an enhanced value compared to the normal state. Such an abrupt conversion was also very recently studied in Ref. 24, where it was demonstrated that the conversion was associated with a transition from pure even-frequency to pure odd-frequency correlations. The simultaneous decrease in the DOS when moving away from the Fermi level results in a rapid decrease in the specific heat, thus leading to the nonmonotonic behavior shown in Fig. 3.

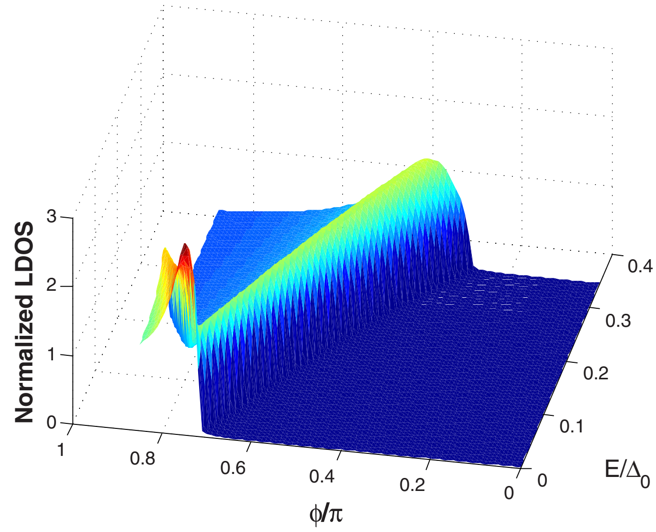

FIG. 4. (Color online) The normalized local density of states of the diffusive S|F|S junction vs phase difference and energy for homogeneous structure of ferromagnetic layer with exchange field $h/\Delta_0=1.1$ and thickness $d_F/\xi_S=0.3$.

IV. SUMMARY

We have demonstrated that the electronic specific heat of the S|F|S junction can be tuned to undergo a strong enhancement by increasing the phase difference between two superconducting leads. The experimental requirement for the observation of this effect is that the width d_F of the ferromagnetic interlayer is considerably smaller than the superconducting coherence length (typically d_F in the range 5–10 nm), and that the exchange field is comparable in magnitude to the gap. The effect persists in the domain-wall case up to exchange fields $h/\Delta_0\simeq 3$, yielding h in the range 4–7 meV for a weak ferromagnetic alloy. Our prediction may be tested experimentally, e.g., by the construction of a SQUID loop geometry[5] by employing calorimetry measurements of an array of S|F|S Josephson junctions.[11]

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